

Preface

In Brazil, I studied with great teachers, and abroad, after a brief contact with Nadia Boulanger in Paris, I received (in the United States) a Master's Degree in Music Theory from Indiana University and a Doctoral Degree in Piano-Performance from Boston University. It was, however, an intense desire of understanding music in its essence that led me to an analysis which is not completely committed to the ways that have been proposed. As a tool, I developed a system that allows the mapping, with the help of graphs, of the most important events in a piece. The graphic representation of musical time in scale allowed me to perceive that, in the works of the great composers, proportions are planned with considerable precision. It was through this system that in 1995 (as a teacher of the *Universidade Estadual de Campinas*) I published the first steps of the subject that is central to this study. In the same way I did then, I am compelled (now in the status of a teacher at the *Universidade Federal da Bahia*) to make public my findings, so that people interested in music may profit from the analytical process I have been developing.

As I cannot include the complete list of all those who offered their hands along my road, I specially acknowledge the two who made this publication possible – Helena Starzynski and Jmary Oliveira. They know why.

Mention must be done, though, of the following names: Jan Schymura for the revision of the text in British English; Fabrizio Messina and Aldo Brizzi for the translation into Italian, and Pablo Sotuyo Blanco for the computer assistance.

The responsibility for both the right and wrong in this article is entirely mine.

Handel's Messiah and Beethoven's op. 110, 123, and 125

Elizabeth Rangel Pinheiro

The purpose of this article is to report the steps that led me to realise structural connections between the oratorio Messiah, written by Handel in 1741,¹ and three representative works by the late Beethoven – Piano Sonata No.31 (1821), Missa Solemnis (1823), and Symphony No.9 (1824).²

The plans Beethoven had for visiting London, at the time he was composing those pieces, might have inspired him to examine Handel's oratorio, a work quite akin to contemporary English musical taste. This was not, however, the clue for my initial understanding. It was the study of proportions in Beethoven's Opus 110 that guided me to the ideas presented here.³ The term 'proportions', as used in this study, refers to the balanced organisation of time in the large-scale of musical structures. In the works in question, the following four structural points usually receive a special treatment: the end of the first third, the middle, the golden section, and the end of the second third. So, the analysis focuses mainly on them.⁴

Although the flexibility which defines the musical excellence of a piece can never be graphically expressed, a system of graphs in scale was devised as a tool to observe how time is organised in the works considered here; the graphs are supported by tables containing calculations related to the duration of the pieces and their main musical divisions, as well as to the main data about their proportions.⁵ The tables follow the tempo changes of each work, according to score indications and recorded performances.⁶ It has been assumed that agogic inflections (fermatas, rubato, cadential rallentandos,

recitative pacing, cadenzas, etc.), suggested by the composer (in the score) or those that naturally occur in performance, do not affect the structure's foundation.

Beethoven's Piano Sonata Opus 110 is in three movements and has A flat major as its tonal centre. The first movement is based on the sonata principle, the second follows a scherzo-trio-scherzo scheme, and the third contains several sections. The sharp contrast between the various portions of the work does not prevent it from displaying a high sense of unity. Although many unifying devices are detectable, only two have to be mentioned at this point. The first is the thematic material that connects the sonata's beginning to the third movement's fugue, mm.[27-114], as well as to the embellished version of the fugal material in the final portion of the work, mm.[174-213]. The second device is the tempo relationship between movements and sections, easily deducible from the indications given by Beethoven. The two initial movements, for example, can be seen as tempo-related, since the crotchet of the first movement's *Moderato cantabile molto espressivo* comfortably equals the semibreve of the second movement's *Allegro molto*. This sort of reasoning for accommodating all the tempo changes allows the piece to be played with a continuous pulse.⁷ The present analysis assumes the following tempo relationships:

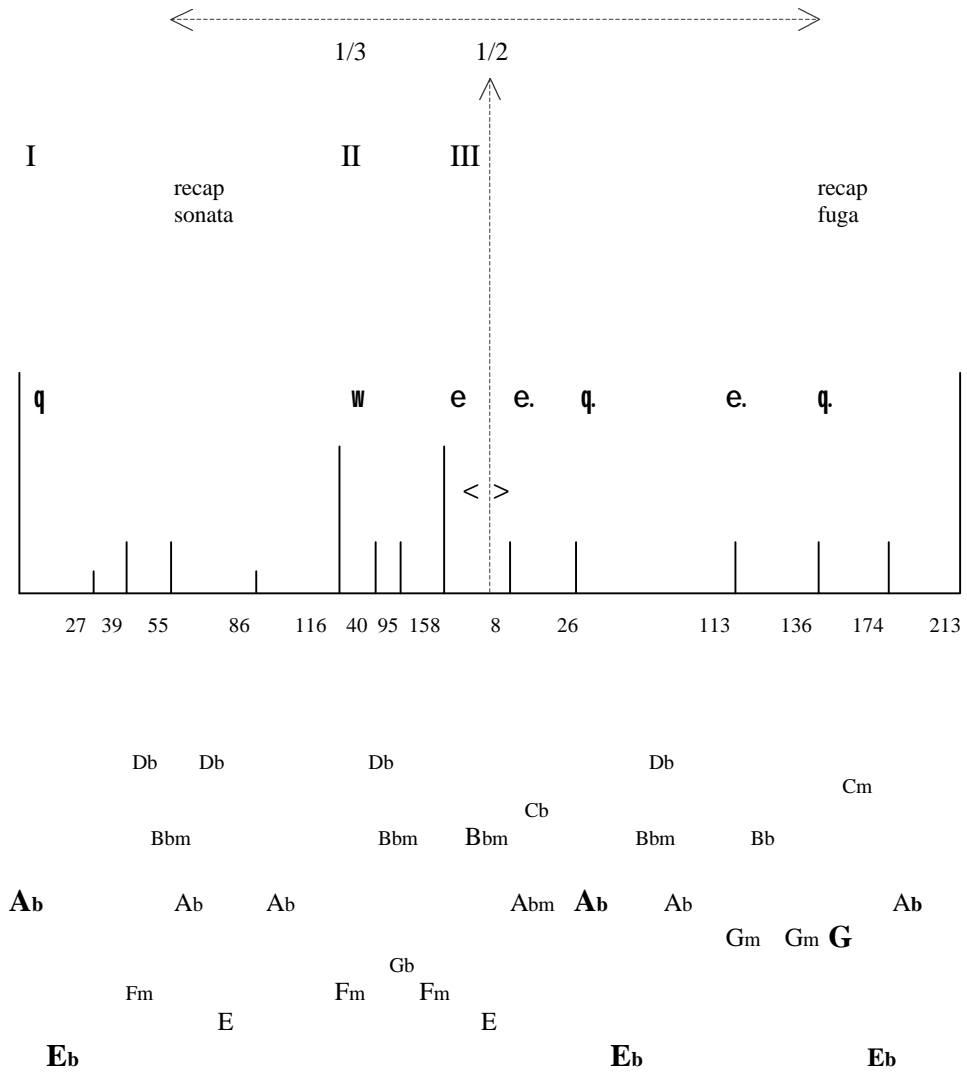
mov.	tempo-character indication	measure	pulse	=
I	<i>Moderato cantabile molto espressivo</i>	[1-116]	crotchet	=
II	<i>Allegro molto</i>	[1-158]	semibreve	=
III	<i>Adagio ma non troppo</i>	[1- 8]	quaver	=
	<i>Arioso dolente</i>	[9- 26]	dotted quaver	=
	<i>Allegro ma non troppo (Fuga)</i>	[27-115]	dotted crotchet	=
	<i>L'istesso tempo di Arioso (Perdendo le forze, dolente)</i>	[116-136]	dotted quaver	=
	<i>L'istesso tempo della Fuga (L'inversione della Fuga)</i>	[137-174]	dotted crotchet	=
	<i>Tempo primo</i>	[174-213]	dotted crotchet	=

An unusual event takes place at m.[5] of the third movement. Not only does the texture differ from the rest of the piece, and two semibreves are placed within a 4/4 measure, but repeated A notes are also given, by Beethoven, an intriguing fingering – the number 3 for a tied note which is not supposed to be played. Reinforced by dynamics through the signs < >, such an unusual event coincides with the middle of the structure (fig. 1). This was the first point related to proportions to attract my attention.

Later I came to realise that three important portions of the piece have their golden sections clearly demarcated: the first movement has a modulation at m.[70] right after the change from the flat to the sharp side of the keys, the second movement has at m.[96] the return of the scherzo after the trio, and the fugue displays the note D^b, at m.[81], as the culmination of an expanded version of the subject which, initiated at m.[73], is enhanced by octaves in *ff*.

Through the observation of each section of the third movement, I noticed that at m.[125], in the golden area of the *Arioso dolente, perdendo le forze*, the 'intriguing fingering' reappears. This reappearance, by occurring within one of the most expressive portions of the work, impelled me to investigate its positioning in relation to the whole of the structure. It was when I found out that the distance of the 'intriguing fingering' back to the middle of the piece is the same which connects the middle back to the change from the flat to the sharp side of the keys, at m.[67] of the first movement. This perception led me, in turn, to realise that the recapitulation of the sonata form in the first movement, at m.[56], is symmetrical to the recapitulation of the fugue, through its inverted version, at

Fig. 1 Graph in scale 1/8 for Beethoven's Opus 110



m.[137] of the third movement. At that time, the only thing I was considering was that Beethoven might have intended a synthesis of the baroque and classical periods, by symmetrically connecting two of their most characteristic compositional devices – fugal writing and sonata form, respectively.⁸

Once awakened, the curiosity about Beethoven's works contemporary to Opus 110 led me to the Missa Solemnis. At the first listening, I was surprised by a quotation of Messiah's Hallelujah,⁹ and first informed through the notes of the recording, that Beethoven had examined Messiah during the long process of composing the Missa (1819-1823).

Owing to a lucky coincidence, I had studied the oratorio right before preparing Beethoven's Opus 110 for performance. As soon as I started working on the sonata's fugue, I noticed a compositional technique that resembles the one used by Handel in the choral fugue *He trusted in God*, No.28 of Messiah.¹⁰ The technique consists in building the counterpoint to the fugal subject mostly in parallel thirds. This procedure had drawn my attention in Messiah because, besides not favouring the voices to be distinct from each other, it is applied, in a consistent way, only to that fugue of the oratorio. Evidence such as this resemblance of the fugues, the presence in the sonata of the term 'arioso' that Handel uses exclusively in No.30 of the oratorio, the presence of recitative, as well as the mystical character of both works, taken together, led me to the insight: 'Beethoven is representing Christ's life; the recapitulation of the fugue, with the words "di nuovo vivente" (living again), stands for Resurrection, and the recapitulation of the first movement's sonata form stands for Nativity.' Later, when returning to a deeper study of Messiah, I would see that what I had noticed through the absolute music of the sonata is

stated by the text in Messiah – *Behold, a virgin shall conceive* of No.8 initiates the 'rite of Nativity' which goes up to the end of No.17, and *Unto which of the angels said* of No.34 initiates the 'rite of Resurrection', which goes up to the end of No.44, the *Hallelujah*. (Both rites are pointed out in the graph of Appendix B.)

Data obtained through a bibliographic search confirmed the ideas I was developing.¹¹ In the collection preserved by the Berlin Royal Library as Artaria 197, the sketches for the three movements of Opus 110 follow the sketches for the *Agnus Dei* of the Missa Solemnis. This assembling, in itself meaningful, increases in significance by the fact that the same collection displays entries in Beethoven's handwriting of the fugue *He trusted in God* of Messiah, the fugue mentioned above.

It was still more surprising to find out that the autograph of Opus 110 carries the equally meaningful date 'December 25, 1821'.¹² Beethoven did not dedicate this sonata as he used to do. Would not Christ be the dedicatee?

The year in this date may raise another question. Would not the composer, through his oratorio-like piano piece, be celebrating the eightieth anniversary of Messiah?

Even though these questions are unanswerable, the awareness created by the findings led me to others. Once, intending to investigate why Beethoven had chosen A flat major as the central key for Opus 110, I examined a facsimile of Messiah's autograph.¹³ I already knew that in the second part of the oratorio, where the events connected to Christ's Passion are presented, all flat keys are activated. So, I was sure that A flat would be found. The great surprise, however, was to discover that when a cadence on A flat occurs, at m.[19] of No.24, the first violin begins a line with the same contour of the subject in the fugue of Beethoven's Opus 110.

In order to evaluate the importance of this A flat event in Messiah, some comments on its structure are necessary. The oratorio contains fifty three pieces divided into three parts. The text of the first part deals with Prophecies and Nativity, the second with Passion and Resurrection, and the third with the spreading and fundamentals of the Christian faith. The scenes in the oratorio mostly follow a recitative-aria-chorus scheme. Thus, the assembling of three choruses – Nos.24, 25, 26, by escaping from that scheme, becomes unavoidably noticeable.¹⁴ Although a simple division of fifty three by two suffices to perceive that the assembling occurs in the central area of the work, the graph in Appendix B may help to observe the placement of the A flat event. Consequently, the question about Beethoven's choice for Opus 110's tonality inadvertently led me to assume that he might have seen how Handel dealt with proportions in his Messiah.¹⁵

From the thirty two sonatas Beethoven wrote for the piano, only Opus 110 displays the word 'vivente.' The closeness of the word to the normal musical indications 'vivo' and 'vivace' may have prevented the literal meaning of the expression 'di nuovo vivente' to be considered in its double role – programmatic and structural. This 'resurrecting' meaning is supported by the event of mm.[131-132] which transforms the G minor of the *Arioso dolente* into the 'luminous' G major that introduces the inverted fugue. In the system mostly employed in Latin cultures, the note G corresponds to 'sol', which in English means 'sun'. Would Beethoven not have wanted to point to this meaning, by associating the Italian words 'di nuovo vivente' with G major? It is worth mentioning that, both in the Missa Solemnis and in the Ninth Symphony, this tonality is used in order to highlight significant points of the text – in the Missa, when the violin solo, at m.[110] of the *Sanctus*, helps the voices (unable to reach very high pitches) to

celebrate the *Benedictus qui venit in nomine Domini*,¹⁶ and in the Ninth, when G major, after m.[595] in the fourth movement, takes the text to the ethereal music for the words '*überm Sternenzelt muss ein lieber Vater wohnen*'.¹⁷

The relationship between Opus 110 and Messiah is reinforced by a motivic aspect that ends up revealing an important connection of the structures of the two works. The second movement in the sonata displays, at mm.[17-20], a motif similar to the one that sets the words *His burthen is light* at mm.[3-4] of No.21 in the oratorio. As can be seen in Example 1, the placement of two short values (quavers in the sonata, and semi-quavers in the chorus) on a strong metrical position gives this motif a rhythmic quality which distinguishes it from all the other motifs. A scherzando-like character pervades both pieces. Both pieces precede the 'Passion' music that, in Messiah is placed in the second part and, in the sonata, can be seen as represented by the *Adagio* at the beginning of the third movement. The idea about the relationship is also reinforced by the rest at mm.[140-141] in the second movement of Opus 110, which appears to emulate the rest Handel often uses before final cadences and that, in No.21, occurs at m.[48].

Ex. 1 a) the right hand at mm.[17-20] of the second movement in Opus 110



b) the soprano line at mm.[3-4] of No.21 in Messiah



His bur-then is light His bur-then, His

The *Adagio* at the beginning of the third movement of Opus 110 displays a 'C' time signature that Beethoven disregards in mm.[4-6]. The freedom of the recitative would have allowed him to use however many crotchets he wanted in m.[4]. Yet, he does it in such a way that the *Adagio* comes to contain thirty three crotchets (including the two quavers of m.[8] which precede the change of texture). To relate this number to Christ's age would seem audacious, were it not for the fact that unquestionably exquisite music is given to the thirty three initial measures of the *Sanctus* in the Missa Solemnis.

After dating the autograph of Opus 110 at the end of 1821, Beethoven reworked the fugue, and mm.[107-109] received a series of sketches (ex. 2 a and b), before reaching the final version (ex. 2 c).¹⁸ It appears that, his aim was to enhance the ending of the subject's last presentation, by making the A flat octave (in the left hand), receive the weight of a downbeat. His careful treatment of this subject version suggested to me that he might have intended to quote the ending of the appealing melodic gesture of the choral entrance in No.4 of Messiah (ex. 2 d). The richness of images suggested by the episodes of Christ's life, as told in the Scriptures, makes it difficult to determine which of them a composer would select for depicting musically. Conscious that programmatic attributions to absolute music are quite subjective, I dare, though, to present my idea on this presumed quotation. By taking the *Arioso dolente, perdendo le forze* in the sonata's third movement as the symbol for Christ's agony, the preceding fugue and its mm.[107-109] can be seen as symbols, respectively, for the *via crucis* and crucifixion. Once the relationship between Opus 110 and Messiah is assumed, it becomes possible to suppose that Beethoven might have intended to recall an important melodic gesture of the oratorio at such a significant moment of the piano piece's 'plot'.

Ex. 2

Different versions toward the final one for mm.[107-109]

a) sketch

Two staves of music in a minor key. The upper staff features a melodic line with eighth and sixteenth notes, including some grace notes. The lower staff provides a harmonic accompaniment with chords and moving lines.

b) sketch

Two staves of music in a minor key. The upper staff has a more rhythmic and chordal texture than sketch a). The lower staff continues with a similar accompaniment style.

c) final version

Two staves of music in a minor key. This version shows a more refined and complex texture, with more intricate chordal structures in both staves.

..... !

d) the alt at mm.[11-14] in No.4 of Messiah

A single staff of music in a major key, showing a melodic line with a mix of eighth and sixteenth notes.

and the glo - ry, the glo- ry of the Lord

..... !

If on the one hand the analysis of Beethoven's Opus 110 guided me to Handel's Messiah, it is necessary to state that, on the other hand, to perceive the proportions in Beethoven's piece was only possible through what I had seen in Messiah. Besides the importance attributed to the middle of the structure already examined, a careful treatment of proportions can be seen in the oratorio. Special events clearly demarcate other important structural points both in relation to the work's whole, and in the context of individual parts, scenes and pieces.

Although the tables in Appendix A make it easy to locate any event in the work, the analysis of the oratorio's proportions depends on its graphic representation. The general criteria for making its graph (presented in Appendix B) have, therefore, to be considered now (details are explained in Appendix A). It is not difficult to perceive, according to Handel's indications, that he conceived structured tempos for relating the pieces in the oratorio. Clear examples are provided by Nos.1 and 2. In No.1, the crotchet of the initial *Grave* equals the minim of the *Allegro moderato*, at m.[13], and a comfortable pacing is obtained with the metronome at 90. Between No.1 and No.2, another kind of relationship is established, since the *larghetto* of No.2 demands the crotchet at 60. These two kinds of relationship – 90 to 90, and 90 to 60 – appear throughout the work. Handel's tempo indications are very logical,¹⁹ and the indication *a tempo ordinario* which appears in Nos.33 and 39 is quite instructive. Since in both pieces a comfortable pacing is obtained with the metronome at 60 for the crotchet, it becomes plausible to deduce that the 'ordinario tempo' was taken as a basic pulse. A pulse that composers could not yet achieve from a metronome, but possibly from the experiments toward the precise determination of a second in the contemporary clocks.

The text, as the basis of the work, offers the best point of departure for considering the oratorio's proportions.²⁰ As seen above, it clarifies the division of the work into three parts. It also clarifies how the composer used the meaning of specific words for musically establishing large-scale connections. 'Lamb', for example, besides linking the many Nativity-related pastoral pieces of the work to the image of Christ as a shepherd, links the symbol for Him as the immolated victim – *Behold the Lamb of God* – at the beginning of the second part, to the oratorio's final statement – *Worthy is the Lamb*. However, it was the word 'glory' that led me to discern the work's overall plan. Although present in Nos. 9, 10, 14, and 53, it is in Nos. 4, 17 and 33 that the word has to be examined. It is emphasised in No.4 – *And the Glory of the Lord* – by the entrance of the chorus, and in No.17, not only through the incisive setting of *Glory to God*, but also through the remarkable contrast created by this incisiveness in relation to the serenity of the previous recitatives. In No.33, the word becomes still more prominent when, to the question *Who is the King of Glory*, repeatedly posed by the double chorus along the piece, the whole chorus gives the assertive answer *He is the King of Glory*. For understanding the structural importance of the word glory, it is necessary, however, to observe motivic and tonal aspects.

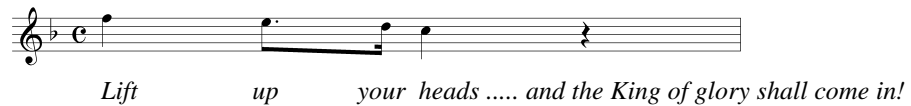
Every piece of Messiah is introduced by an unmistakable motif. Yet, the initial motifs of Nos.17 and 33 resemble each other. Although belonging to different tonal areas, both describe a descending gesture with the notes F-E-D-C in rhythmic correspondence (ex. 3). While in No.17, the gesture sets the word glory itself, in No.33, it is used for introducing the *King of Glory* text.

Ex. 3

a) Soprano line at m.[1] in Messiah's No.17



b) Soprano line at m.[5] in Messiah's No.33



The musical importance of the 'glory' motif is confirmed by tonality. In No.17, the motif appears in the D major context of the piece, and its presence in No.33 precedes the D major of Nos.34-35. This motivic/tonal relationship is enhanced by the fact that No.17 closes the 'rite of Nativity', and the 'rite of Resurrection' begins right after the assertive answer *He is the King of Glory*, at the end of No.33. As the graph in Appendix 2 shows, the rites act as D major prolongations of corresponding lengths.²¹

Indeed, the oratorio contains three D major prolongations: Nativity, Resurrection and the final portion, from *The trumpet shall sound* to the end. They are contrasted by a prolongation of the flat side of the keys that, initiated in No.18 where B flat replaces the previous D major, ends with the change from the F major of No.33 to the D major of Nos.34-35. The return to the sharp side of the keys comes to be seen as quite relevant, once proportions are considered. The text in Nos. 35 and 36 – *Let all angels worship Him* and *Thou art gone up on high* - makes a clear reference to Christ's Ascension. As if intending to celebrate the top meaning of the text, the composer devotes the best of his contrapuntal craftsmanship to the D-major centred No.35.²² And through the smoothness characteristic of minor tonalities, he imprints on the D-minor centred No.36, the serene

mood adequate to the moment. This major-minor D gathering of Nos.35 and 36 coincides with the golden area of the whole work.

Coincidentally or not, the above described relationships (D major/B flat major and D major/D minor) are the same that appear in the large-scale tonal plans of Beethoven's Opus 123 and Opus 125 (see the graphs in Appendix B). In the Missa Solemnis, the D major/B flat relationship is symmetrically defined, since the B flat of the *Credo* is preceded and followed by the D major which predominates in the other Mass' items. In the Ninth Symphony, B flat major is displayed as the main source of tonal contrast to the minor-major-D tonality upon which the structure is built. Besides defining the symmetrical arrangement of the work by acting as the tonal centre of the third movement, B flat acts as the tonal area both for the second thematic material in the exposition of the first movement's sonata form, and for the *Alla Marcia* at m.[331] of the fourth movement.

A great balance exists between the first part of Messiah, represented by 9,07 centimetres in the graph of Appendix B, and the second part, represented by 9,26 centimetres (18,33 - 9,07). It is impossible to guess which steps were followed in the compositional process. Yet from the score, it becomes possible to make conjectures about the size of the third part as having been intended for placing the important Passion-related choruses Nos.24-25-26 in the middle of the structure, and Christ's Ascension in the golden area of the work.

While the previous comments refer to the larger context of the oratorio, the following refer to the proportional arrangement in the context of the parts and individual pieces. The most significant example is offered by the chorus *For unto us a child is born* which, meaningfully written in G major, begins in the golden section of the first part. Nos.4 and 53 can exemplify the proportional arrangement in the context of the individual

pieces. Written in A major, No.4 has, at m.[83], an F sharp chord strongly emphasised both by the first violin at the top of the texture, and by the meeting of the four choral voices in the word 'together'. Introducing the only presentation in B major (the dominant-of-the-dominant) of the initial text-thematic material, this chord (the most distant from the tonic in the piece) takes place exactly in the golden area. In No.53, the golden area coincides with the conspicuous texture created, at m.[92], by the entrance of the single violin line presenting the subject of a fugal event. The importance of this entrance resides in the fact that fugal expositions exclusively performed by the violins occur only twice in the whole work: here and in mm.[13-21] of No.1. (Would Beethoven not have wanted to emulate the importance of this proportional placement of the violin, by giving the instrument a distinctive role both in the *Benedictus* of the Missa Solemnis and in the third movement of the Ninth Symphony?)

Religiosity, seen in this study as the main link for the whole net of works, might have been, in Beethoven's case, brought about through the involvement he had with Mass' subjects alongside the work on the Missa Solemnis. Regarded by him as his best work,²³ Opus 123 is in fact a repository of the composer's craftsmanship at a highly developed stage. Any attempt at making a synthesis of such a complex work would fail. So, only the aspects connected to the central purpose of this study will be dealt with.

As seen previously, it was a quotation of Messiah's Hallelujah that provided me with the first clue for perceiving the relationship between Handel's oratorio and Beethoven's Opus 110 and Opus 123. It was another quotation from Messiah that made me change the plans I had had of developing a consistent aural acquaintance with the Missa, before facing the analysis of the score. At the first listening, a musical event strongly drew my attention, which impelled me to look at the score. The moving power of

the *Gloria's* beginning is suddenly interrupted at m.[43] by a unison played with soft dynamics. The effect immediately reminded me of the one Handel creates in No.17, in the 'Nativity rite', when the moving power built for the text *Glory to God in the highest* is suddenly interrupted, at m.[5], by a unison with soft dynamics. With the scores in hand, I realised that the soft unison on the pitch A sets the same text in both pieces – *and peace on earth* in Messiah, and *et in terra pax* in the Missa.

The quotations of Messiah reveal an important aspect of proportions in the Missa. As the graph for Opus 123 in Appendix B shows,²⁴ the two quotations (the exclamation marks) are presented in connection with the word 'peace'.²⁵ The placement of the word exhibits the balanced arrangement of the structure, since the 1,90 centimetre from the beginning of the work to the appearance of the word *pax*, at m.[43] in the *Gloria*, is balanced by the 1,53 centimetre of the *Dona nobis pacem* which closes the Missa.

Beethoven sets the five items that constitute the Mass ordinary (as used in the majority of the events throughout the year in the Catholic Liturgy). This number of items is in itself an appeal for symmetrical plans, since the long text of the *Credo*, preceded by the *Kyrie* and *Gloria* and followed by the *Sanctus* and *Agnus Dei*, naturally acts as a structural centre. Yet, it is not in this general sense that Beethoven proceeds to apply the principle of symmetry to Opus 123. By daring to alter the inviolability of the liturgical text,²⁶ he changes the beginning of the statement *Et in Spiritum Sanctum*, to make the word *Credo* part of the recapitulation created, at m.[264], by the return of the B flat gesture of the *Credo's* beginning. This recapitulation is set right in the middle of the structure. Still more important to the main purpose of the present study is to observe that this B flat is preceded and followed by the D major events of mm.[240 and 279,

respectively]. This juxtaposition of B flat and D major in the middle of the structure, besides reflecting the work's large-scale tonal plan, reaffirms the relationship between the Missa Solemnis and Messiah.

The symmetrical arrangement in Opus 123 is also revealed by the corresponding distance between important events placed at each side of the work's middle point. For example, the distance of 0,85 centimetre between *Et resurrexit* at m.[188] and the middle is balanced by the 0,87 centimetre between the middle and *Et vitam venturi* at m.[306]. (The size of each portion, extracted from the table in Appendix A, can be observed in the graph of Appendix B). The clearest example is given, though, by the corresponding sizes of the *Kyrie* (1,74 centimetre) and the *Dona nobis pacem* (1,53 centimetre), respectively, at the beginning and at the end of the work.

The entrance of the violin solo at m.[110] in the *Sanctus* is often the focus of comments on Opus 123, because this unusual insertion of the violin (in a solo role) into the liturgical context of the Mass integrates the list of Beethoven's daring compositional innovations. As previously commented, the G major tonality of the violin-solo section, apparently intended to represent 'light', connects the text of the *Benedictus* both to the Nativity event in Messiah, and to the 'Resurrection' inverted fugue in Beethoven's Opus 110. Yet, there are two aspects that still have to be mentioned. One is related to proportions, since the *Benedictus* initiates the third third of the work. The other is the adoption by Beethoven of the time signature 12/8 which gives the *Benedictus* section the same bucolic atmosphere that pervades the pastoral pieces in Messiah.

The thirty three measures that, at the beginning of the *Sanctus*, set with a silent prayer (*Mit Andacht* as Beethoven suggests) the introduction to the core of the Mass' rite – the Eucharist – were, apparently, conceived to coincide with the golden section of the

work. Even if the tempos of the many sections are differently chosen in performance, due to the long span of the *Adagio* measures, the golden section still occurs within them.²⁷ Various factors contribute to make this portion of Opus 123 stand out. One is the tonal colour change promoted by the return of D major, the work's central tonality, after the B-flat-centred *Credo*. A change, however, that Beethoven smoothes with the initial pitch B acting as a chromatic bridge between the *Credo*'s final B flat and the D major sonority of m.[2] in the *Sanctus*. Equally smoothed is the harmonic progression that, guided by the bass line, moves carefully thus preventing angular leaps or strong cadences to disturb the serenity of the musical pace. It is this smooth moving pace that leads to the dominant pedal of mm.[28-33] where, helped by the tremolo in *pp* of the strings and timpano in A, the music reaches one of the most ethereal moments in the work.

Beethoven summarises the interplay of the minor and major modes in D, which supports the structure of the Ninth Symphony, in the fourth-movement's recitative of mm.[216-236]. For these twenty-one measures that are part of one of the most remarkable changes in music history – the entrance of the human voice into the symphonic genre, he becomes a poet for inviting Man to Schiller's invitation to Joy – 'O friends, not these tones! Rather, let us begin to sing more pleasant and more joyful ones' (as Beethoven's words are translated in the Dover's score). Through these measures, the composer explicitly demonstrates the character differentiation he attributed to minor and major modes – the third of D minor, which sets the word 'Töne' with its implied reference to sadness, is replaced by the third of D major welcoming joy in '*anstimmen, und freudenvollere!*'.

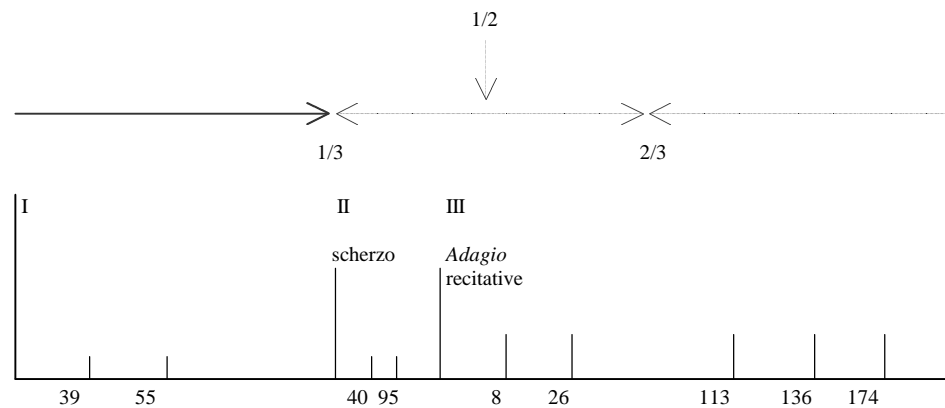
The overall plan of Opus 125 is quite close to the one Beethoven uses in Opus 110. Preceded by the vivacity characteristic of scherzos, and followed by a sequence of sections, the slow movement occupies a central structural position in both works (see the graphs in fig. 2). Corresponding compositional styles are featured by the sections after the slow portions, and in this correspondence, *recitative* has the prior role. The *ariosos* demanding the piano to 'sing' grief in Opus 110, are replaced by the vocal music 'playing' joy in the Ninth. Fugal writing, which constitutes the very embryonic idea of Opus 110, besides permeating the whole texture of the Ninth, is taken to represent, through the vitality of the orchestral fugal portion after the fourth movement's *Alla marcia*, the enthusiastic appeal to 'victory' of the text 'Happily as His suns fly through heaven's splendid field, run your course, brothers, joyfully as a hero to victory' (as translated in the Dover score). So, in the same way that in Opus 110 a balance is established by the placement of the scherzo and fuga at each side of the central *Adagio*, in the Ninth, the vitality of the second movement's scherzo is balanced by the vivacious pace of the heroic *Alla marcia*. Whereas this balance reveals the underlying principle of symmetry in a quite subtle way, the division into thirds of both structures reveals it quite openly: in Opus 110, the first third contains the first movement, and in the Ninth, the third third contains the fourth movement (the division into thirds is indicated in fig. 2).

Various stages were passed through as I studied proportions in the Ninth Symphony. It had been easy, from the beginning, to notice the proportional construction of each movement. Unequivocal is, for example, the placement of the trio at m.[412] and the scherzo's return at m.[531] demarcating, respectively, the middle and the golden section of the second movement (exactly the same procedure adopted in Opus 110).

Fig. 2 Overall plan of Opus 110 in scale **1/8**, compared to the overall plan of Opus 125 in scale **1/32**

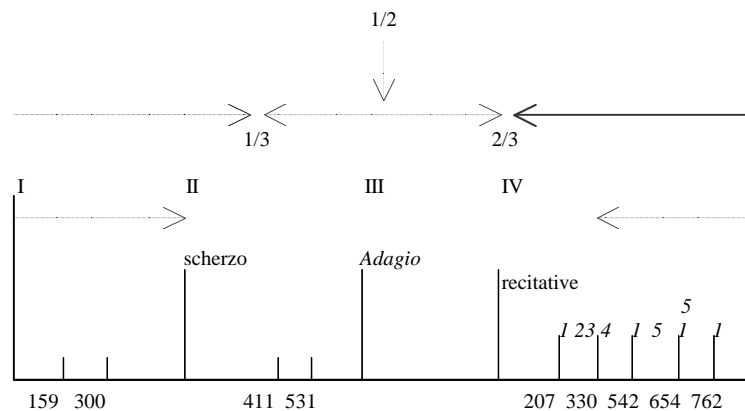
Opus 110

1/8 scale



Opus 125

1/32 scale



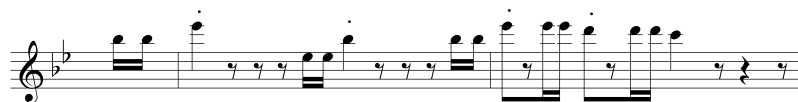
Concerning the whole, the precision of measurements that had been observed in Opus 110 could not be found. I was attributing this imprecision to the complexity of writing for diversified media – orchestra, chorus and soloists, added to the problems of text setting. Then, suddenly, I was informed that recent research had revealed the correct metronome for the *Alla Marcia* – 84 applied to the minim and not to the crotchet, as it had traditionally been done.²⁸ This information urged me to redo the graph I had prepared before, and with the *Alla marcia* reduced to a half, I came to realise that, also in this work, the whole is governed by very precise proportions.

The new graph led me to a point when I began to see things through Brahms's eyes. Continuously involved with this composer's music, I was at that time examining his First Symphony.²⁹ The relationship between this work and Beethoven's Ninth (not denied by Brahms) is well known. What the study with the graphs' aid revealed, however, was that the relationship does not concern only the always mentioned transference of Beethoven's vocal-choral idea to the orchestral choral which appears at m.[61] of the fourth movement in Brahms's First. By comparing the two works' proportions, it becomes possible to notice that, more than a simple quotation, Brahms created deep structural links between them.

The comparison of the two works' structures has to begin with a close observation of the third movement in Beethoven's Ninth. It is organized as a series of variations on two themes: one presented at the beginning, and the other at m.[25]. While the latter has a clear reappearance at m.[65], the former is treated in a more complex way. At m.[3], it appears in the top voice of a choral-like texture. At m.[43], the embellished line of the first violin sets it as an accompanied melody. After m.[83], it receives quite a special

treatment. Fragmented in the wood-winds, it dialogs with the prominent solo attributed to the horn. The accompaniment by the strings in *pizzicato*, besides creating a rarefied texture that contributes to the prevailing atmosphere of expectation, subtly prepares the ternary beat subdivision of the following section. While the middle of the movement coincides with the beginning of the specially treated music of mm.[83-98], the golden section coincides with the beginning of the variation in 12/8. Here, the thematic material in the first violin, capriciously elaborated by diminution technique, is transformed into a musical lace. It is necessary to observe that from the beginning of the movement, the level of dynamics was to be kept soft. Although indications of *cresc.* exist, they suggest only nuances to the large number of *ps* and *pps* that Beethoven includes, as if he wanted to say that nothing should disturb the *Adagio molto e cantabile* quality of the movement. Then, at m.[121], an effect which evokes the idiomatic calls of brass instruments is suddenly suggested for the whole ensemble in *f* unison. Inserted in the golden area of the movement, this brass-like call coincides, in turn, with the golden section of the whole work. It would have been impossible to evaluate the importance of this event without Brahms's help. In his First Symphony, the golden section, at m.[30] of the fourth movement, is demarcated by the prominent horn-call theme shown in Example 4.³⁰ In the graphs of Figure 3, the calls are represented by exclamation marks.

Ex. 4 'Brass' calls a) mm.[121-122] in the third movement of Beethoven's Ninth Symphony



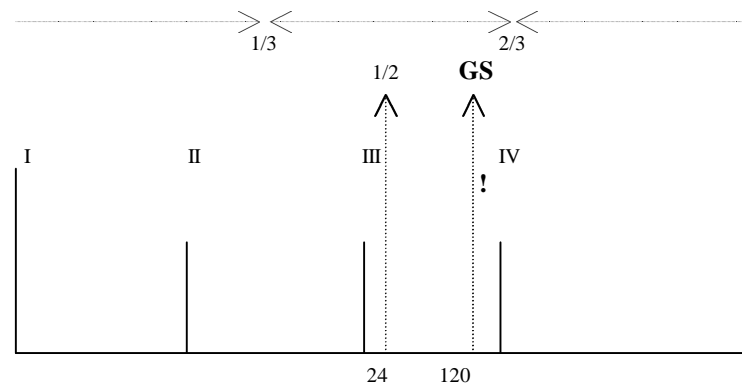
b) mm.[30-38] in the fourth movement of Brahms's First Symphony



Fig. 3 Overall plan of Beethoven's Opus 125 compared to the overall plan of Brahms's Opus 68 (both in scale 1/32)

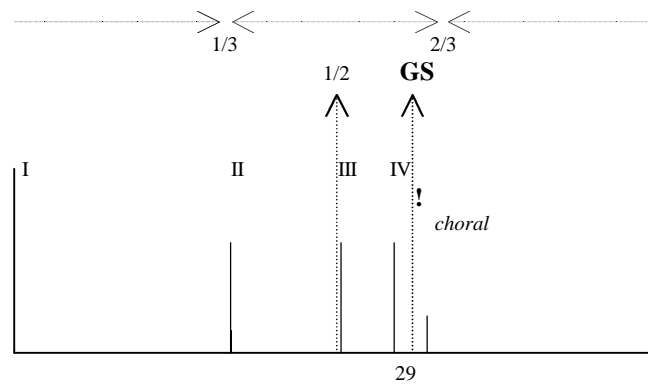
Beethoven's Opus 125

1/32 scale



Brahms's Opus 68

1/32 scale



The above assertion about the golden section in Brahms's First is mainly based on the clarity which prevails in the organisation of the work. From the way the composer signals the two halves and the three thirds, it can be deduced that he was dealing with precise mathematical ratios. As the graph shows (fig.3), the third movement begins in the middle of the structure, and the thirds contain, respectively: 1) the first movement, 2) the second and third movements, as well as the fourth movement's sixty initial measures, and 3) the musical portion which begins with the instrumental choral.

The game between binary and ternary rhythmic components, known as hemiola, has often been associated with Brahms's name. What he does in the First Symphony is nothing more than a structural hemiola, the same kind of hemiola he might have perceived in Beethoven's Ninth. Games between binary and ternary components seem to have been one of the most efficient means used by composers for adding flexibility to the musical flow. In Opus 110, Beethoven creates several layers of such games. In the first movement, at m.[5], the game is between two dotted crotchets and three crotchets, while from mm.[44-49], at each two measures, a dotted minim plays with three minims, as the pins < > for dynamics clarify. In Messiah, hemiola is not only a common trait in cadential points, but the game between binary and ternary components appears in the construction of entire portions as well. The initial *ritornello* in No.52 serves as an interesting example. Its twenty-four measures result from the superposition of differentiated layers. The ternary meter (3/4) creates one such layer. Significant harmonic changes and melodic figurations allow one to see that the portion consists of eight blocks of three measures each. It is upon the flexibility of this binary-versus-ternary game that Handel applies the hemiolic flexibility of mm.[7-8 and 23-24]. Another example of hemiola, here clarified

by the text, is offered by No.4, since the syllables 'be' and 'veal' are stressed by rhythmic values, as shown in Example 5.

Ex. 5 Messiah's No.4, mm.[33-38]

and the glo - ry, the glo - ry of the Lord shall **be re - veal - ed**

It was Brahms again who shed light on the most recent stage of my studies on this whole net of works. The choral melody in Beethoven's Ninth emphasises the intervals of the octave, the fifth and the fourth, the same intervals basic to the main thematic material of the whole work. There is, though, another interval that is emphasised in the choral melody. It is the ascending sixth A-F sharp stressed by syncopation (see mm.[103-104 and 111-112] of the fourth movement in Example 6). Two other impressive traits of the choral theme are the rhythmic figure 'dotted crotchet/quaver', and the neighbour note that embellishes the pitch F sharp. From a close observation of the 'choral melody' in Brahms's First, it becomes easy to notice that it contains the same distinguishing traits as Beethoven's choral melody – neighbour quaver embellishments, syncopation and ascending leaps of a sixth. In Brahms's work, D tonal areas are consistently avoided. Indeed, only in the final measures, a D minor sonority appears in the emphatic choral texture of mm.[407-416]. According to my understanding, Brahms conceived a tonal link

between the two works, and made it clear through the dominant-tonic relationship that exists from the Ninth's choral theme to the initial melodic gesture G-C of his 'choral' theme. The assembling of the choral melodies in Example 6 results from this understanding.

Ex. 6 Assembled choral melodies of Beethoven's Ninth, mm.[92-115] in the fourth movement, and Brahms's First, mm.[62-78] in the fourth movement.

The image displays five staves of musical notation. The first three staves are in bass clef with a key signature of one sharp (F#) and a common time signature (C). The first two staves show a melodic line with a mix of quarter and eighth notes, including some rests. The third staff continues this line and ends with a double bar line. The fourth and fifth staves are in treble clef with a key signature of one flat (Bb) and a common time signature (C). They show a melodic line with a mix of quarter and eighth notes, including some rests. The fifth staff ends with a double bar line.

Although apparently unrelated to the core of this study, it was the above understanding that urged me to compare the nature of the basic intervals in the thematic material of Beethoven's Ninth to the ones used by Handel in Messiah. Easily noticeable is the way Handel exploits the expressive power of the intervals to benefit the text. The feeling of anguish, for example, is expressed by the tritone in *Thy rebuke hath broken His heart* in No.29, or by the diminished seventh in *And with His stripes* in No.25. The most important point, however, is the way he uses the perfect intervals of the octave, the fourth, and the fifth (mostly integrating tonic sonorities). Avoided as initial motifs for the pieces from No.1 to No.21, the impulse of the ascending octave is reserved for the

lamenting *Behold the Lamb of God* in No.22, and for one of the most expressive moments of the whole work, when the choral bass, from mm.[76-92] in No.26, states: '*and the Lord laid on Him, the iniquity of us all*'. To the fifth and its inversion, the fourth, Handel attributes not only the majority of initial motifs, but also the most important motivic roles. Significant examples of the ascending fourth appear in Nos.23, 24, 26, 35 and 36, while the descending fourth is represented by the 'glory' motifs of Nos.17 and 33, as well as by Nos.41, 44 and 53. The ascending fifth that delimits the tonic triad is present in many initial gestures; No.4 offers the clearest example. Impressing is, though, the treatment Handel gives to the descending fifth. It links the pieces in Nos.6, 7, 9 and 12, pieces that deal mostly with the tenderness appropriate to the 'rite of Nativity'. I dare to present my idea that Beethoven might have desired to transfer the importance of the intervals of the fifth, the fourth and the octave in Messiah to his Ninth Symphony. The octave on the pitch A that encompasses the choral theme represents the sum of the descending fifth of the initial phrase (see mm.[92-99] in ex. 6) and the descending fourth from the tonic D down to A in the second phrase (mm.[100-107] in ex.6). These are the intervals that appear as the motivic basis for the most important melodies of the work. The following examples may suffice. A descending fifth introduces the work, while an ascending fifth introduces the fourth movement's recitative. A descending fourth introduces the third movement. (The rhythmically emphasised pitches D and G in the melody that, at m.[414], introduce the second movement's *Presto* was purposely chosen to exemplify the ascending fourth, because Brahms subtly quotes this melody at m.[118] of the third movement of his First Symphony.) The octave is conspicuously exemplified, in Beethoven's Ninth, by the gesture that introduces the second movement.

It is largely known that Brahms examined the works of his predecessors with great care and he, obviously, knew the Messiah. Also known is his position of admitting the direct relationship between his First Symphony and Beethoven's Ninth. What he might not have imagined, however, is that his work could come to be seen as indirectly connected to Handel's oratorio. As for Beethoven, he might have been sure that the connection between his works and Messiah would come to be noticed. What both him and Handel possibly did not imagine is that their works, the Ninth and Messiah, would become official hymns of human fraternity 'forever and ever'.

APPENDIX A

Tables for the graphs of

Handel's	<u>Messiah</u>
Beethoven's	<u>Opus 110</u>
	<u>Opus 123</u>
	<u>Opus 125</u>
Brahms's	<u>Opus 68</u>

About the tables

The calculations in the tables consider millimetres and take three decimals as their point of departure. This choice was not without problems. First of all, it would have been impossible to depart from centimetres employing two decimals (as the computer program used for the graphs does) because a number such as 0.031 millimetre ($1.000 \div 32$) would disappear when transformed into centimetre. Another problem came from the following. If four decimals are used, $0.5000 \div 8 = 0.0625$ for example, the calculation is more precise than in a calculation using three: $0.500 \div 8 = 0.062$. While in the former, the millimetre is achieved precisely ($0.0625 \times 16 = 1.000$), the result of the latter is 0.992 (0.062×16). The solution was to disregard this difference (0.008 millimetre) because, besides being minuscule, it is not considered by the computer in the making of the graphs. Thus, the final decision was to present all the tables in the same format: the calculations employ millimetres with three decimals, and the two right-hand columns which correspond to the scales $1/8$ and $1/32$ follow the computer reading in centimetres with two decimals.

Concerning Messiah, the tempo choices are based on the Dover score, Gardiner's recording, and my own experience with the work. Yet, some remarks are due.

- In the *Andante* of No.14, Gardiner uses the same tempo as in the *Allegro* of No.16. In the table, the crotchet is, though, considered at 60 in the *Andante*, and at 90 in the *Allegro*.
- For No.13, Gardiner uses the existing long version, while in the table the short Dover version is considered (comments on the *Pifa's* two versions in Mann, p.217).

- The Dover version is considered in No.18.
- The unaccompanied recitatives (which according to baroque practices were supposed to be free) do not carry tempo indications. Apparently intended as bridges, they mostly serve to prepare the tempo of the following piece. It is on this idea that the tempos in the tables are based.
- Mann (p.213) states: 'Initial tempo indications are not consistently capitalised in the autograph; *adagio* (invariably in lower case) is used in the autograph only to indicate a concluding *ritenuto*'. Although structured tempos can be applied between the main body of the pieces and their cadential *adagios*, they are considered in the tables according to Mann's statement. Only the *adagio* at mm.[76-92] of No.26 is taken as a real change of tempo.
- For No.40, the Dover version is considered (comments on the two versions of this piece in Mann, p.223).

Concerning Beethoven's Opus 125, a remark is due. According to Jonathan del Mar in the Critical Edition (Bärenreiter, p.38), 'the true metronome mark of the Trio (mm.412-530) is irretrievably lost'. He suggests 160 for the minim, and his suggestion was adopted for the table.

The tables inform the tempo changes, the main formal divisions, and the points of structural significance mentioned in the text.

Table for the graphs of Beethoven's Piano Sonata, Opus 110

scale		1/1		1/8	
a second	metronome	millimetre		beat	x 3
60	÷ 60	= 1.000		÷ 8 = 0.125	0.375
60	÷ 120	= 0.500		÷ 8 = 0.062	

			{Computer <i>Word</i> reading}		
	metronome	measures	millimetre	centimetre	centimetre
				scale 1/8	scale 1/32

I

q	60 [1 - 19] = 19 x 0.375	= 7.125	0,71	0,17
	60 [20 - 27] = 8 x 0.375 = 3.000 + 7.125	= 10.125	1,01	0,25
	60 [28 - 39] = 12 x 0.375 = 4.500 + 10.125	= 14.625	1,46	0,36
	60 [40 - 55] = 16 x 0.375 = 6.000 + 14.625	= 20.625	2,06	0,51
	60 [56 - 69] = 14 x 0.375 = 5.250 + 20.625	= 25.875	2,58	0,64
	60 [70 - 78] = 9 x 0.375 = 3.375 + 25.875	= 29.250	2,92	0,73
	60 [79 - 86] = 8 x 0.375 = 3.000 + 29.250	= 32.250	3,22	0,80
	60 [87 - 116] = 30 x 0.375 = 11.250 + 32.250	= 43.500	4,35	1,08

II

h	120 [1 - 40b] = 80 x 0.062 = 4.960 + 43.500	= 48.460	4,84	1,21
	120 [41 - 95] = 55 x 0.062 = 3.410 + 48.500	= 51.870	5,18	1,29
	120 [96 - 158] = 95 x 0.062 = 5.890 + 51.870	= 57.760	5,77	1,44

III

e	60 [1 - 8] = 72 x 0.125 = 9.000 + 57.760	= 66.760	6,67	1,66
e.	60 [9 - 26] = 71 x 0.125 = 8.875 + 66.760	= 75.635	7,56	1,89
q.	60 [26 - 114] = 176 x 0.125 = 22.000 + 75.635	= 97.635	9,76	2,44
e.	60 [114 - 136] = 88 x 0.125 = 11.000 + 97.635	= 108.635	10,86	2,71
q.	60 [136 - 174] = 76 x 0.125 = 9.500 + 108.635	= 118.135	11,81	2,95
q.	60 [174 - 213] = 78 x 0.125 = 9.750 + 118.135	= 127.885	12,78	3,19

Main data about proportions in Beethoven's Opus 110

Scale 1/8 centimetre

		1/3	1/2	GS	2/3
TOTAL	= 12,78	4,26	6,39	7,89	8,52
I	= 4,35	1,45	2,17	2,68	2,90
II	= 1,42	0,47	0,71	0,87	0,94
III	= 7,01	2,33	3,50	4,33	4,66

sections of III

Adagio	= 0,90	0,30	0,45	0,55	0,60
Arioso	= 0,88	0,29	0,44	0,54	0,58
Fuga	= 2,20	0,73	1,10	1,35	1,46
Arioso	= 1,10	0,36	0,55	0,67	0,72
Inv. Fuga	= 0,95	0,31	0,47	0,58	0,62
Finale	= 0,97	0,32	0,48	0,59	0,64

Scale 1/32 centimetre

		1/3	1/2	GS	2/3
TOTAL	= 3,19	1,06	1,59	1,97	2,12
I	= 1,08	0,36	0,54	0,66	0,72
II	= 0,36	0,12	0,18	0,22	0,24
III	= 1,75	0,58	0,87	1,08	1,16

sections of III

Adagio	= 0,22	0,07	0,11	0,13	0,14
Arioso	= 0,22	0,07	0,11	0,13	0,14
Fuga	= 0,55	0,18	0,27	0,33	0,36
Arioso	= 0,27	0,09	0,13	0,16	0,18
Inv. Fuga	= 0,23	0,07	0,11	0,14	0,14
Finale	= 0,24	0,08	0,12	0,14	0,16

Table for the graphs of Handel's Messiah

scale a second	metronome	1/1 millimetre	1/8 per beat	x 2	x 3	x 4
60	÷ 30	= 2.000 ÷ 8 = 0.250		0.500	0.750	1.000
60	÷ 45	= 1.333 ÷ 8 = 0.166		0.333	0.499	0.666
60	÷ 60	= 1.000 ÷ 8 = 0.125		0.250	0.375	0.500
60	÷ 75	= 0.800 ÷ 8 = 0.100		0.200	0.300	0.400
60	÷ 90	= 0.666 ÷ 8 = 0.083		0.166	0.249	0.333
60	÷ 105	= 0.571 ÷ 8 = 0.071		0.142	0.214	0.285
60	÷ 120	= 0.500 ÷ 8 = 0.062		0.125	0.187	0.250
60	÷ 135	= 0.444 ÷ 8 = 0.055		0.111	0.166	0.222

	metron.	measures	millimetre	{ Computer <i>Word</i> reading }	centimetre scale 1/8	centimetre scale 1/32
PART I						
1	q	90 [1 - 12] = 12 x 0.333	= 3.996		0,39	0,09
	q	90 [1 - 12] = 12 x 0.333 = 3.996 + 3.996 = 7.992			0,79	0,19
	h	90 [13 - 97] = 85 x 0.166 = 14.110 + 7.992 = 22.102			2,21	0,55
2	q	60 [1 - 37] = 37 x 0.500 = 18.500 + 22.102 = 40.602			4,06	1,01
3	h	60 [1 - 84] = 84 x 0.250 = 21.000 + 40.602 = 61.602			6,16	1,54
4	h.	60 [1 - 138] = 138 x 0.125 = 17.250 + 61.602 = 78.852			7,88	1,97
5	q	90 [1 - 30] = 30 x 0.333 = 9.990 + 78.852 = 88.842			8,88	2,22
6	e	90 [1 - 58] = 58 x 0.249 = 14.442 + 88.842 = 103.284			10,32	2,58
	h	90 [59 - 93] = 35 x 0.166 = 5.810 + 103.284 = 109.094			10,90	2,72
	e	90 [94 - 114] = 21 x 0.249 = 5.229 + 109.094 = 114.323			11,43	2,85
	h	90 [115 - 150] = 36 x 0.166 = 5.976 + 114.323 = 120.299			12,02	3,00
	h	90 [151 - 158] = 8 x 0.249 = 1.992 + 120.299 = 122.291			12,22	3,05
7	q	90 [1 - 58] = 58 x 0.333 = 19.314 + 122.291 = 141.605			14,16	3,54
8	q	60 [1 - 6] = 6 x 0.500 = 3.000 + 141.605 = 144.605			14,46	3,61
9	q.	60 [1 - 150] = 150 x 0.250 = 37.500 + 144.605 = 182.105			18,21	4,55
10	q	60 [1 - 23] = 23 x 0.500 = 11.500 + 182.105 = 193.605			19,36	4,84
11	q	60 [1 - 63] = 63 x 0.500 = 31.500 + 193.605 = 225.105			22,51	5,62
12	q	90 [1 - 99] = 99 x 0.333 = 32.967 + 225.105 = 258.072			25,80	6,45
13	q.	60 [1 - 11] = 11 x 0.500 = 5.500 + 258.072 = 263.572			26,35	6,58

14	q 60 [1 - 4] = 4 x 0.500 = 2.000 + 263.572 = 265.572	26,55	6,63
	q 60 [5 - 11] = 7 x 0.500 = 3.500 + 265.572 = 269.072	26,90	6,72
15	q 60 [1 - 9] = 9 x 0.500 = 4.500 + 269.072 = 273.572	27,35	6,83
16	q 90 [1 - 8] = 8 x 0.333 = 2.664 + 273.572 = 276.236	27,62	6,90
17	h 60 [1 - 49] = 49 x 0.250 = 12.250 + 276.236 = 288.486	28,84	7,21
18	q 120 [1 - 108] = 108 x 0.250 = 27.000 + 288.486 = 315.486	31,54	7,88
19	q 90 [1 - 8] = 8 x 0.333 = 2.664 + 315.486 = 318.150	31,81	7,95
20	q. 60 [1 - 56] = 56 x 0.500 = 28.000 + 318.150 = 346.150	34,61	8,65
21	q 90 [1 - 51] = 51 x 0.333 = 16.983 + 346.150 = 363.133	36,31	9,07

PART II

22	q 45 [1 - 32] = 32 x 0.666 = 21.312 + 363.133 = 384.445	38,44	9,61
23	q 45 [1 - 49] = 49 x 0.666 = 32.634 + 384.445 = 417.079	41,70	10,42
	q 45 [50 - 67] = 18 x 0.666 = 11.988 + 417.079 = 429.067	42,90	10,72
	q 45 [1 - 49] = 49 x 0.666 = 32.634 + 429.067 = 461.701	46,17	11,54
24	q 45 [1 - 26] = 26 x 0.666 = 17.316 + 461.701 = 479.017	47,90	11,97
25	h 90 [1 - 91] = 91 x 0.166 = 15.106 + 479.017 = 494.123	49,41	12,35
26	h 60 [1 - 75] = 75 x 0.250 = 18.750 + 494.123 = 512.873	51,28	12,82
	h 60 [76 - 92] = 17 x 0.500 = 8.500 + 512.873 = 521.373	52,13	13,03
27	q 60 [1 - 11] = 11 x 0.500 = 5.500 + 521.373 = 526.873	52,68	13,17
28	h 60 [1 - 63] = 63 x 0.250 = 15.750 + 526.873 = 542.623	54,26	13,56
29	q 45 [1 - 18] = 18 x 0.666 = 11.952 + 542.623 = 544.797	54,47	13,61
30	q 45 [1 - 15] = 15 x 0.666 = 9.990 + 544.797 = 554.787	55,47	13,86
31	q 90 [1 - 5] = 5 x 0.333 = 1.665 + 554.787 = 556.452	55,64	13,91
32	q 90 [1 - 43] = 43 x 0.333 = 14.319 + 556.452 = 570.771	57,07	14,26
33	h 60 [1 - 77] = 77 x 0.250 = 19.250 + 570.771 = 590.021	59,00	14,75
34	q 90 [1 - 5] = 5 x 0.333 = 1.665 + 590.021 = 591.686	59,16	14,79
35	q 90 [1 - 37] = 37 x 0.333 = 12.321 + 591.686 = 604.007	60,40	15,10
36	q 90 [1 - 116] = 116 x 0.249 = 28.884 + 604.007 = 632.891	63,28	15,82

37	q 90 [1 - 25] = 25 x 0.333 = 8.325 + 632.891 = 641.216	64,12	16,03
38	q. 45 [1 - 24] = 24 x 0.666 = 17.984 + 641.216 = 659.200	65,92	16,48
39	h 60 [1 - 38] = 38 x 0.250 = 9.500 + 659.200 = 668.700	66,87	16,71
40	q 135 [1 - 45] = 45 x 0.222 = 9.990 + 668.700 = 678.690	67,86	16,96
41	q 135 [1 - 67] = 67 x 0.166 = 11.122 + 678.690 = 689.812	68,98	17,24
42	q 105 [1 - 4] = 4 x 0.285 = 1.140 + 689.812 = 690.952	69,09	17,27
43	q 105 [1 - 74] = 74 x 0.214 = 15.836 + 690.952 = 706.788	70,67	17,66
44	q 105 [1 - 94] = 94 x 0.285 = 26.790 + 706.788 = 733.578	73,35	18,33

PART III

45	q 90 [1 - 164] = 164 x 0.249 = 40.836 + 733.578 = 774.414	77,44	19,36
46	q 60 [1 - 6] = 6 x 0.500 = 3.000 + 774.414 = 777.414	77,74	19,43
	h 60 [7 - 16] = 10 x 0.250 = 2.500 + 777.414 = 779.914	77,99	19,49
	q 60 [17 - 22] = 6 x 0.500 = 3.000 + 777.914 = 782.914	78,29	19,57
	h 60 [23 - 37] = 15 x 0.250 = 3.750 + 782.914 = 786.664	78,66	19,66
47	q 60 [1 - 8] = 8 x 0.500 = 4.000 + 786.664 = 790.664	79,06	19,76
48	q 120 [1 - 155] = 155 x 0.187 = 28.985 + 790.664 = 819.649	81,96	20,49
	q 120 [156 - 213] = 58 x 0.187 = 10.846 + 819.649 = 830.495	83,04	20,76
	q 120 [29 - 156] = 128 x 0.187 = 23.936 + 830.495 = 854.431	85,44	21,36
49	q 75 [1 - 5] = 5 x 0.400 = 2.000 + 854.431 = 856.431	85,64	21,41
50	q 75 [1 - 24] = 24 x 0.400 = 9.600 + 856.431 = 866.031	86,60	21,65
51	q 75 [25 - 74] = 50 x 0.400 = 20.000 + 866.031 = 886.031	88,60	22,15
52	q 120 [1 - 178] = 178 x 0.187 = 33.286 + 886.031 = 919.317	91,93	22,98
53	q 60 [1 - 7] = 26 x 0.125 = 3.250 + 919.317 = 922.567	92,25	23,06
	h 60 [7 - 11] = 9 x 0.125 = 1.125 + 922.567 = 923.692	92,36	23,09
	q 60 [12 - 19] = 30 x 0.125 = 3.750 + 923.692 = 927.442	92,74	23,18
	h 60 [19 - 23] = 9 x 0.125 = 1.125 + 927.442 = 928.567	92,85	23,21
	q 90 [24 - 71] = 48 x 0.333 = 15.984 + 928.567 = 944.551	94,45	23,61
	q 90 [72 - 159] = 88 x 0.333 = 29.304 + 944.551 = 973.855	97,38	24,34

Main data about proportions in Handel's Messiah

Scale 1/8		centimetre			
		1/3	1/2	GS	2/3
TOTAL	= 97,38	32,46	48,69	60,18	64,92
Part 1	= 36,31	12,10	18,15	22,43	24,20
Part 2	= 37,04	12,34	18,52	22,89	24,68
Part 3	= 24,03	8,01	12,01	14,85	16,02

Scale 1/32		centimetre			
		1/3	1/2	GS	2/3
TOTAL	= 24,34	8,11	12,17	15,04	16,23
Part 1	= 9,07	3,02	4,53	5,60	6,05
Part 2	= 9,26	3,08	4,63	5,72	6,17
Part 3	= 6,00	2,00	3,00	3,71	4,00

Table for the graphs of Beethoven's Missa Solemnis, Opus 123

scale a second	metronome	1/1 millimetre	1/8 per beat	x 2	x 3	x 4	x 6
60	÷ 48	= 1.250 ÷ 8	= 0.156	0.312	0.468	0.624	0.936
60	÷ 60	= 1.000 ÷ 8	= 0.125	0.250	0.375	0.500	0.750
60	÷ 66	= 0.909 ÷ 8	= 0.113	0.226	0.339	0.452	0.678
60	÷ 72	= 0.833 ÷ 8	= 0.104	0.208	0.312	0.416	0.624
60	÷ 80	= 0.750 ÷ 8	= 0.093	0.186	0.279	0.372	0.558
60	÷ 90	= 0.666 ÷ 8	= 0.083	0.166	0.249	0.332	0.498
60	÷ 100	= 0.600 ÷ 8	= 0.075	0.150	0.225	0.300	0.450
60	÷ 120	= 0.500 ÷ 8	= 0.062	0.124	0.186	0.248	0.372

metronome	measures	millimetre	{ Computer <i>Word</i> reading } centimetre scale 1/8	centimetre scale 1/32	
<i>KYRIE</i>					
h	48	[1 - 85] = 85 x 0.312 + 0.156	= 26.676	2,66	0,66
h	72	[86 - 127] = 42 x 0.312 = 13.104 +	26.676 = 39.780	3,97	0,99
h	48	[128 - 223] = 96 x 0.312 = 29.952 +	39.780 = 69.732	6,97	1,74
<i>GLORIA</i>					
h.	48	[1 - 130] = 130 x 0.156 = 20.280 +	69.732 = 90.012	9,00	2,25
h.	48	[131 - 173] = 43 x 0.156 = 6.708 +	90.012 = 96.720	9,67	2,41
h.	48	[174 - 229] = 56 x 0.156 = 8.736 +	96.720 = 105.456	10,54	2,63
e	60	[230 - 309] = 80 x 0.500 = 40.000 +	105.456 = 145.456	14,54	3,63
h.	48	[310 - 359] = 50 x 0.156 = 7.800 +	145.456 = 153.256	15,32	3,83
h	60	[360 - 458] = 99 x 0.250 = 24.750 +	153.256 = 178.006	17,80	4,45
h	80	[459 - 524] = 66 x 0.186 = 12.276 +	178.006 = 190.282	19,02	4,75
h.	72	[525 - 569] = 45 x 0.104 = 4.680 +	190.282 = 194.962	19,49	4,87
<i>CREDO</i>					
h	72	[1 - 123] = 123 x 0.208 = 25.584 +	194.962 = 220.546	22,05	5,51
q	60	[124 - 143] = 20 x 0.500 = 10.000 +	220.546 = 230.546	23,05	5,76
q	72	[144 - 155] = 12 x 0.312 = 3.744 +	230.546 = 234.290	23,42	5,85

e	72	[156 - 187] =	32 x 0.624 =	19.968 + 234.290 =	254.258	25,42	6,35
h	72	[188 - 193] =	6 x 0.208 =	1.248 + 254.258 =	255.506	25,55	6,38
w	72	[194 - 263] =	70 x 0.104 =	7.280 + 255.506 =	262.786	26,27	6,56
h	72	[264 - 305] =	42 x 0.208 =	8.736 + 262.786 =	271.522	27,15	6,78
h	80	[306 - 372] =	67 x 0.279 =	18.693 + 271.522 =	290.215	29,02	7,25
h	120	[373 - 432] =	60 x 0.186 =	11.160 + 290.215 =	301.375	30,13	7,53
h	100	[433 - 472] =	40 x 0.450 =	18.000 + 301.375 =	319.375	31,93	7,98

SANCTUS

e	60	[1 - 33] =	33 x 0.500 =	16.500 + 319.375 =	335.875	33,58	8,39
h	60	[34 - 52] =	19 x 0.250 =	4.750 + 335.875 =	340.625	34,06	8,15
h.	60	[53 - 78] =	26 x 0.125 =	3.250 + 340.625 =	343.875	34,38	8,59
q	60	[79 - 110] =	32 x 0.375 =	12.000 + 343.875 =	355.875	35,58	8,89
q.	60	[111 - 234] =	124 x 0.500 =	62.000 + 355.875 =	417.875	41,78	10,44

AGNUS DEI

q	60	[1 - 95] =	95 x 0.500 =	47.500 + 417.875 =	465.375	46,53	11,63
q.	66	[96 - 163] =	68 x 0.226 =	15.368 + 465.375 =	480.743	48,07	12,01
h	90	[164 - 189] =	26 x 0.166 =	4.316 + 480.743 =	485.059	48,50	12,12
q.	72	[190 - 265] =	76 x 0.208 =	15.808 + 485.059 =	500.867	50,08	12,52
w	90	[266 - 353] =	88 x 0.083 =	7.304 + 500.867 =	508.171	50,81	12,70
q.	66	[354 - 434] =	81 x 0.226 =	18.306 + 508.171 =	526.477	52,64	13,16

Main data about proportions in Beethoven's Missa Solemnis, Opus 123

Scale 1/8	centimetre		1/3	1/2	GS	2/3
TOTAL	= 52,64		17,54	26,32	32,53	35,08
<i>Kyrie</i>	= 6,97		2,32	3,48	4,30	4,64
<i>Gloria</i>	= 12,52		4,17	6,26	7,73	8,34
<i>Credo</i>	= 12,44		4,14	6,22	7,68	8,28
<i>Sanctus</i>	= 9,85		3,28	4,92	6,08	6,56
<i>Agnus Dei</i>	= 10,86		3,62	5,43	6,71	7,24

Scale 1/32	centimetre		1/3	1/2	GS	2/3
TOTAL	= 13,16		4,38	6,58	8,13	8,77
<i>Kyrie</i>	= 1,74		0,58	0,87	1,07	1,16
<i>Gloria</i>	= 3,13		1,04	3,13	1,93	2,08
<i>Credo</i>	= 3,11		1,03	1,55	1,92	2,07
<i>Sanctus</i>	= 2,46		0,82	1,23	1,52	1,64
<i>Agnus Dei</i>	= 2,71		0,90	1,35	1,67	1,81

Table for the graphs of Beethoven's Ninth Symphony, Opus 125

scale	1/1	1/8				
a second	metronome	millimetre	per beat	x 2	x 3	x 4
60	÷ 60	= 1.000 ÷ 8	= 0.125	0.250	0.375	0.500
60	÷ 63	= 0.952 ÷ 8	= 0.119	0.238	0.357	0.476
60	÷ 66	= 0.909 ÷ 8	= 0.113	0.226	0.339	0.452
60	÷ 72	= 0.833 ÷ 8	= 0.104	0.208	0.312	0.416
60	÷ 80	= 0.750 ÷ 8	= 0.093	0.186	0.279	0.372
60	÷ 84	= 0.714 ÷ 8	= 0.089	0.178	0.267	0.356
60	÷ 88	= 0.681 ÷ 8	= 0.085	0.170	0.255	0.340
60	÷ 96	= 0.625 ÷ 8	= 0.078	0.156	0.234	0.312
60	÷ 116	= 0.517 ÷ 8	= 0.064	0.128	0.192	0.256
60	÷ 120	= 0.500 ÷ 8	= 0.062	0.124	0.186	0.248
60	÷ 132	= 0.454 ÷ 8	= 0.056	0.112	0.168	0.224
60	÷ 160	= 0.375 ÷ 8	= 0.046	0.092	0.138	0.184

metronome measures	millimetre	{Computer <i>Word</i> reading}	
		centimetre scale 1/8	centimetre scale 1/32
I			
q 88 [1 - 73] = 73 x 0.170 = 12.410	= 12.410	1,24	0,31
[74 - 159] = 86 x 0.170 = 14.620 + 12.410	= 27.030	2,70	0,67
[160 - 217] = 58 x 0.170 = 9.860 + 27.030	= 36.890	3,68	0,92
[218 - 300] = 83 x 0.170 = 14.110 + 36.890	= 51.000	5,10	1,27
[301 - 338] = 38 x 0.170 = 6.460 + 51.000	= 57.460	5,74	1,43
[339 - 426] = 88 x 0.170 = 14.960 + 57.460	= 72.420	7,24	1,81
[427 - 547] = 121 x 0.170 = 20.570 + 72.420	= 92.990	9,29	2,32
II			
h. 116 [1 - 411] = 794 x 0.064 = 50.816 + 92.990	= 143.806	14,38	3,59
h 160 [412 - 530] = 195 x 0.092 = 17.940 + 143.806	= 161.746	16,17	4,04
h. 116 [531 - 941] = 411 x 0.064 = 26.304 + 161.746	= 188.050	18,80	4,70
h 160 [942 - 954] = 13 x 0.092 = 1.196 + 188.050	= 189.246	18,92	4,73
III			
q 60 [1 - 24] = 24 x 0.500 = 12.000 + 189.246	= 201.246	20,12	5,03
[25 - 42] = 18 x 0.357 = 6.426 + 201.246	= 207.672	20,76	5,19

	[43 - 64] =	24 x 0.500 =	12.000 + 207.672	= 219.672	21,96	5,49
	[65 - 82] =	18 x 0.357 =	6.426 + 219.672	= 226.098	22,60	5,65
	[83 - 98] =	16 x 0.500 =	8.000 + 226.098	= 234.098	23,40	5,85
q.	60 [99 - 157] =	59 x 0.500 =	29.500 + 234.098	= 263.598	26,35	6,58

IV

h.	116 [1 - 29] =	29 x 0.113 =	3.277 + 263.598	= 266.875	26,68	6,67
q	88 [30 - 37] =	8 x 0.170 =	1.360 + 266.875	= 268.235	26,82	6,70
h.	66 [38 - 47] =	10 x 0.113 =	1.130 + 268.235	= 269.365	26,93	6,73
h.	116 [48 - 55] =	8 x 0.064 =	0.512 + 269.365	= 269.877	26,98	6,74
h.	66 [56 - 62] =	7 x 0.113 =	0.791 + 269.877	= 270.668	27,06	6,76
q	60 [63 - 64] =	2 x 0.500 =	1.000 + 270.668	= 271.668	27,16	6,79
h.	66 [65 - 76] =	12 x 0.113 =	1.356 + 271.668	= 273.024	27,30	6,82
h	80 [77 - 80] =	4 x 0.186 =	0.744 + 273.024	= 273.768	27,37	6,84
h.	66 [81 - 91] =	11 x 0.113 =	1.243 + 273.768	= 275.011	27,50	6,87
h	80 [92 - 205] =	114 x 0.186 =	21.204 + 275.011	= 296.215	29,62	7,40
h.	80 [206 - 207] =	2 x 0.186 =	0.372 + 296.215	= 296.587	29,65	7,41
h.	66 [208 - 215] =	8 x 0.113 =	0.904 + 296.587	= 297.491	29,74	7,43
h.	66 [216 - 236] =	21 x 0.113 =	2.373 + 297.491	= 299.864	29,98	7,49
h	80 [237 - 330] =	94 x 0.186 =	17.484 + 299.864	= 317.348	31,73	7,93
h.	84 [331 - 542] =	212 x 0.089 =	18.868 + 317.348	= 336.216	33,62	8,40
h.	84 [543 - 594] =	52 x 0.089 =	4.628 + 336.216	= 340.844	34,08	8,52
h	72 [595 - 626] =	32 x 0.312 =	9.984 + 340.844	= 350.828	35,08	8,77
h	60 [627 - 654] =	28 x 0.375 =	10.500 + 350.828	= 361.328	36,13	9,03
h.	84 [655 - 762] =	108 x 0.178 =	19.224 + 361.328	= 380.552	38,05	9,51
h	120 [763 - 850] =	88 x 0.124 =	10.912 + 380.552	= 391.464	39,14	9,78
h	132 [851 - 915] =	65 x 0.112 =	7.280 + 391.464	= 398.744	39,87	9,96
q	60 [916 - 919] =	4 x 0.375 =	1.500 + 398.744	= 400.244	40,02	10,00
w	88 [920 - 940] =	21 x 0.085 =	1.785 + 400.244	= 402.029	40,20	10,05

Main data about proportions in Beethoven's Ninth Symphony, Opus 125

Scale 1/8		centimetre			
		1/3	1/2	GS	2/3
TOTAL	= 40,20	13,40	20,10	24,84	26,80
I	= 9,29	3,09	4,64	5,74	6,18
II	= 9,63	3,21	4,81	5,95	6,42
III	= 7,43	2,47	3,71	4,59	4,94
IV	= 13,85	4,61	6,92	8,55	9,22

Scale 1/32		centimetre			
		1/3	1/2	GS	2/3
TOTAL	= 10,05	3,35	5,02	6,21	6,70
I	= 2,32	0,77	1,16	1,43	1,54
II	= 2,40	0,80	1,20	1,48	1,60
III	= 1,85	0,61	0,92	1,14	1,23
IV	= 3,46	1,15	1,73	2,13	2,30

Table for the graph of Brahms's First Symphony, Opus 68

scale		1/1		1/8			
a second	metronome	millimetre		beat	x 2	x 3	x 4
60	÷ 48	1.250	÷ 8	= 0.156	0.312	0.468	0.624
60	÷ 72	0.833	÷ 8	= 0.104	0.208	0.312	0.416
60	÷ 96	0.625	÷ 8	= 0.078	0.156	0.234	0.312
metronome measures				millimetre	{ Computer reading }		
					centimetre scale 1/8	centimetre scale 1/32	
I							
e 96	[1 - 37]	= 225 x 0.078 =		= 17.550	1,75	0,43	
q. 96	[38 - 120]	= 166 x 0.078 =	12.948 +	17.550 = 30.498	3,04	0,76	
	[121 - 188]	= 136 x 0.078 =	10.608 +	30.186 = 41.106	4,11	1,02	
	[189a -188]	= 302 x 0.078 =	23.556 +	41.106 = 64.662	6,46	1,61	
	[189b -260]	= 144 x 0.078 =	11.232 +	64.662 = 75.894	7,58	1,89	
	[261 - 342]	= 164 x 0.078 =	12.792 +	75.894 = 88.686	8,86	2,21	
	[343 - 393]	= 102 x 0.078 =	7.956 +	88.686 = 96.642	9,66	2,41	
	[394 - 457]	= 128 x 0.078 =	9.984 +	96.642 = 106.626	10,66	2,66	
	[458 - 494]	= 74 x 0.078 =	5.772 +	106.626 = 112.398	11,23	2,80	
q. 48	[495 - 511]	= 17 x 0.312 =	5.304 +	112.398 = 117.702	11,77	2,94	
II							
q 48	[1 - 38]	= 38 x 0.468 =	17.784 +	117.702 = 135.486	13,54	3,38	
	[39 - 66]	= 28 x 0.468 =	13.104 +	135.486 = 148.590	14,85	3,71	
	[67 - 104]	= 38 x 0.468 =	17.784 +	148.590 = 166.374	16,63	4,15	
	[105 - 128]	= 24 x 0.468 =	11.232 +	166.374 = 177.606	17,76	4,44	
III							
q 96	[1 - 70]	= 70 x 0.156 =	10.920 +	177.606 = 188.526	18,85	4,71	
q. 96	[71-108b]	= 60 x 0.156 =	9.360 +	188.526 = 197.886	19,78	4,94	
q 96	[109 - 114]	= 6 x 0.156 =	0.936 +	197.822 = 198.822	19,88	4,97	
	[115 - 153]	= 39 x 0.156 =	6.084 +	198.822 = 204.906	20,49	5,12	
	[154 - 164]	= 11 x 0.156 =	1.716 +	204.906 = 206.622	20,66	5,16	

IV

q	48	[1 - 29] = 29 x 0.624 = 18.096 + 206.622	= 224.718	22,47	5,61
q	72	[30 - 61] = 32 x 0.416 = 13.312 + 224.718	= 238.030	23,80	5,95
q	96	[62 - 117] = 56 x 0.312 = 17.472 + 238.030	= 255.502	25,55	6,38
		[118 - 185] = 68 x 0.312 = 21.216 + 255.502	= 276.718	27,67	6,91
		[186 - 248] = 63 x 0.312 = 19.656 + 276.718	= 296.374	29,63	7,40
		[249 - 301] = 53 x 0.312 = 16.536 + 296.374	= 312.910	31,29	7,82
		[302 - 366] = 65 x 0.312 = 20.280 + 312.910	= 333.190	33,31	8,32
		[367 - 390] = 23 x 0.312 = 7.176 + 333.190	= 340.366	34,03	8,50
h	96	[391 - 457] = 67 x 0.156 = 10.452 + 340.366	= 350.818	35,08	8,77

Main data about proportions in Brahms's First Symphony, Opus 68

Scale 1/8 centimetre

		1/3	1/2	GS	2/3
TOTAL	= 35,08	11,69	17,54	21,67	23,38
I	= 11,77	3,92	5,88	7,27	7,84
II	= 5,99	1,99	2,99	3,70	3,98
III	= 2,90	0,96	1,45	1,79	1,92
IV	= 14,42	4,80	7,21	8,91	9,60

Scale 1/32 centimetre

		1/3	1/2	GS	2/3
TOTAL	= 8,77	2,92	4,38	5,41	5,84
I	= 2,94	0,98	1,47	1,81	1,96
II	= 1,49	0,49	0,74	0,92	0,98
III	= 0,72	0,18	0,36	0,44	0,36
IV	= 3,60	1,20	1,80	2,22	2,40

APPENDIX B

Graphs on the scale of 1/32

Handel's Messiah

and

Beethoven's

Opus 110

Opus 123

Opus 125

About the graphs

The main horizontal line represents the work's duration.

The various lengths of the vertical lines are intended to represent the hierarchical organisation of the work into movements (or parts), sections and sub-sections.

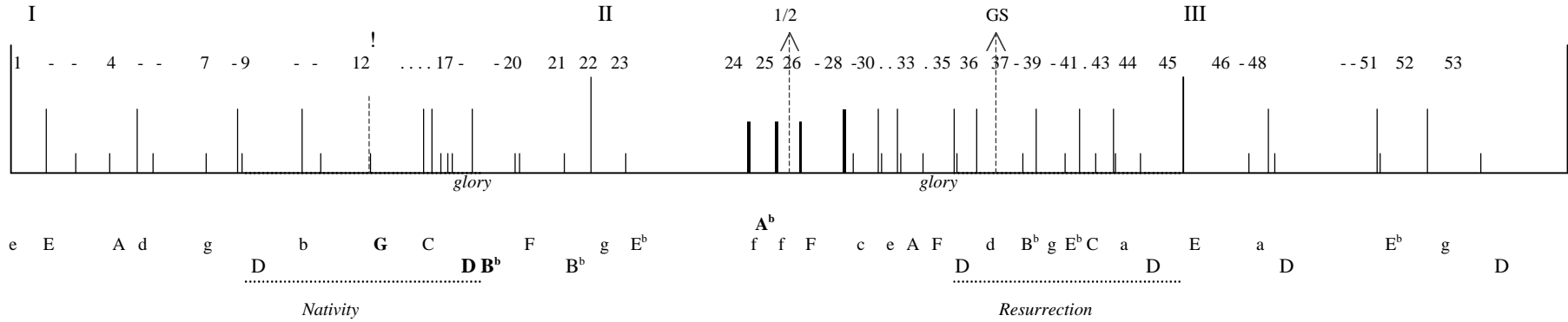
In Messiah's graph, the 53 piece numbers are horizontally displayed above the horizontal line.

In Opus 123 and Opus 125, the numbers attached to the horizontal line refer to the measures within each movement.

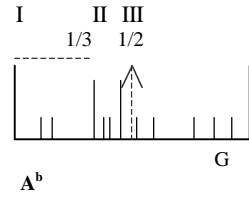
Due to the reduced size of Opus 110's graph on the scale of 1/32, details were omitted. They appear, though, in the graph on the scale of 1/8 within the text.

Tonal references appear below the horizontal line.

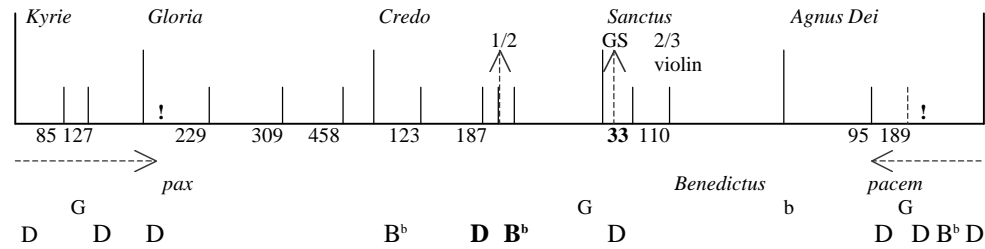
Messiah



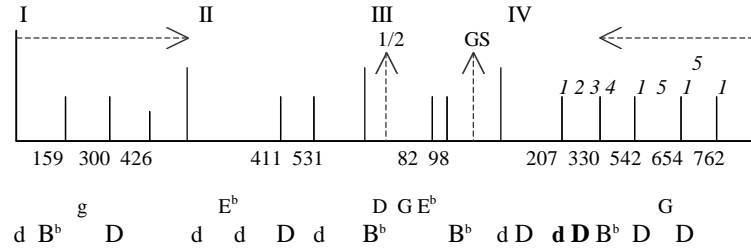
Opus 110



Opus 123



Opus 125



NOTES

¹ The autograph of the original version of Messiah is preserved by the British Library as RM 20.f.2. Fundamental to the present work, regarding the oratorio's history, were Alfred Mann's introduction and the critical notes in the Dover score edition 0-486-26067-4 (1989), as well as Donald Burrows's Handel Messiah (Cambridge, 1991).

² The study is based on the following score editions: Dover (already mentioned) for Handel's Messiah, Henle (1980) for Beethoven's Opus 110, Dover 0-486-26894-2 (1991) for Beethoven's Missa Solemnis, and Bärenreiter BA 9009 (1996) for Beethoven's Ninth Symphony. (The scores should be examined during the reading of this article.)

³ The first steps of this study were published in Brazil (Editora da UNICAMP, 1995) as Proporções no Opus 110 de Beethoven (Proportions in Beethoven's Opus 110).

⁴ In Debussy in Proportion: A Musical Analysis (Cambridge, 1983), p. 2, Roy Howat defines: 'Golden section is the way of dividing a fixed length in two so that the ratio of the shorter portion to the longer equals the ratio of the longer to the entire length.' The golden section can be found by multiplying a given length by 0.618. The appliance of this mathematical ratio to music is a subject under investigation, and some questions still remain to be answered. One of them has to do with the closeness of the golden section to the second third of a given size – 0.666. According to the tripartite exposition-development-recapitulation principle that mostly governs music, special effects used in golden areas can also be understood as preparations for the return of the pieces' initial materials at the beginning of their third thirds. Another question is related to the common practice among baroque composers of basing their musical discourse on rhetorical directions (as mentioned in 1739 by Johann Mattheson in his Der vollkommene Capellmeister). According to such directions, the idea, after being introduced and developed, should at a certain point reach a peak of eloquence, before being taken to a conclusion. In music, the rhetorical peak would correspond to what can be taken as the golden section. Nevertheless, after observing the precision of measurements in the music of baroque composers, I came to believe that the appliance of the golden section principle was taken for granted among them. That this ancient principle, already known in the pre-Christian Greek classical era, attended the highly mathematical thinking of the baroque period is testified, for example, by Johannes Kepler who, living from 1571 to 1630, referred to the presently called golden section as *sectio divina* (Encyclopedia Britannica, 1971, p. 542). In Johann Sebastian Bach's music, the golden section is often demarcated by especial events. The fugue in E flat minor in the first volume of Das Wohltemperierte Klavier clearly illustrates the treatment given to proportions by the composer. The only appearance of the subject in the piece's lowest register uses the weight of a descending fifth (in the heading motif of the subject's inverted version) to emphasise the D sharp tonic, right in the middle of the structure. To the golden section, the composer devotes a quite elaborate event. A canon involving all the voices develops the first half of the subject at mm.[52-53], but at m.[54], the canon changes to develop the first half of the inverted subject. This change occurs exactly in the golden section. (The study of this fugue and its corresponding prelude was in great part responsible for my interest in proportions; the analysis which resulted from this study appears in <http://www.svn.com.br/urucungo>.) Concerning Bach's music, the use of mathematical ratios can be easily deduced from the composer's involvement with the experiments toward the equal temperament. Concerning Beethoven's music, it was the analysis of Opus 110 that made me strongly believe he was dealing with the golden section, and that his practice might have been influenced by the baroque music which, as documented, he closely observed. Fundamental to the present study is, however, the idea that in Opus 110 Beethoven emulates the ways Handel, in Messiah, treats the golden sections both in small and large-scale contexts.

⁵ As the system was conceived, the metronome beat at 60 (one second) would be represented by one millimetre on a scale of 1/1, and the tempo changes calculated as in the following example: 60 ÷ metronome 84 = 0.714 millimetre. The large dimension of Handel's Messiah demanded, however, the use of a reduced scale to fit a manageable publishing size. So, the oratorio's graph appears on the scale of 1/32 (the beat at 60 = 0.031 millimetre) in Appendix B, where it is followed by the other works' graphs in the same scale, in order to make the comparison of their dimensions easy. For allowing details to be shown, the graph that accompanies the discussion of Beethoven's Opus 110 (through which the system is explained) is

on the scale of 1/8 (the beat at 60 = 0.125 millimetre). The tables that served as the basis for the graphs appear in Appendix A.

⁶ Details about the chosen tempos precede the tables in Appendix A, but the sources for them must be mentioned here. For Beethoven's Opus 110, the choices were determined by the listening experience achieved from many live and recorded performances added to my own performing experiments. For the ensemble works, score indications were combined with the tempos adopted in recorded versions under John Eliot Gardiner (Philips 11041 for Messiah, ARCHIV 429779-2 for the Missa Solemnis, and ARCHIV 439905-2 for the Ninth Symphony). After a comparative study of various recorded versions, Gardiner's were chosen because the adopted tempos are closer to the scores' indications than those adopted in the other versions. For Messiah, the tempos which appear in the tables, more than on recorded versions, relied on the analytical work I developed with the musicians who, in 1992, were in charge of preparing live performances under Brazilian conductor Benito Juarez.

⁷ The metronome beat at 60 allows for such continuity. At the performer's will, a faster or slower tempo can be chosen, providing that the tempo relationships are kept. Quite adequate to the present study is the expression 'structured tempos', applied to such relationships by David Epstein; his ideas on time-related subjects appear in Beyond Orpheus (Cambridge, MA, 1979) and Shaping Time (New York, 1995).

⁸ Beethoven's search for relating the sonata form of the first movement to the fugue is revealed by the modulation to E flat major, at m.[45] of the third movement, which allows the fugue exposition in A flat major, from m.[27] to m.[40], to be wholly transferred to the dominant area, from m.[45] to m.[66]. This way, the fugue comes to mimic the tonal areas of the first and second thematic materials in the exposition of the first movement's sonata form – A flat and E flat, respectively (as highlighted in fig.2).

⁹ The quotation, at m.[216] of the *Agnus Dei*, acts as the subject of the *Dona nobis pacem* fugal setting.

¹⁰ The numbering of the pieces in Messiah is not the same in the several versions the oratorio received. In the Dover edition (used in this study), the numbering corresponds to the order that Burrows calls 'S' (Op. cit., pp. 86-100).

¹¹ Data of historical nature related to Beethoven and his works were compiled from: Georg Kinsky, Das Werk Beethovens. Thematisch-bibliographisches Verzeichnis seiner sämtlichen vollendeten Kompositionen, Hans Halm ed. (München, 1955), Thayer's Life of Beethoven, Elliot Forbes ed. (Princeton, 1967), Douglas Johnson, Alan Tyson and Robert Winter, The Beethoven Sketchbooks, Douglas Johnson ed. (University of California Press, 1985), and The Beethoven Compendium, Barry Cooper ed. (London, 1991).

¹² The autograph of the original version is preserved as Artaria 196 by the Berlin Royal Library (Staatsbibliothek Preussischer Kulturbesitz). The date it carries, as discussed in The Beethoven Sketchbooks (Op. cit., pp. 267-70), refers to the day the autograph was concluded.

¹³ The facsimile (London, 1868) gave me an idea about the means Handel might have employed to control the dimensions of the work. Some pages (those corresponding to repeated portions) contain only the bar-lines. Each page is divided into the same number of measures which appear in the neighbouring pages where the music is written down. By multiplying the number of measures by the number of pages, he would have been able to always know at which point of the structure he was, and possibly, through a similar procedure, to control the dimension of the whole, as well. Later, this reasoning received further clarification by Burrows's comments (Op. cit., p. 8) on Handel's method of working: 'With the libretto already arranged into recitatives, arias and (for oratorios) choruses, the first step was to lay out the whole score.....The overall scheme was thereby committed to paper, and the musical shape and tonality of the concerted movements (arias, accompanied recitatives, choruses) established.'

¹⁴ Mann describes (Op. cit., p. 220) the connecting devices: 'Nos. 24, 25, 26 are joined in the autograph: the first change of tempo (*alla breve moderato*) occurs in the middle of a line, the final section (*allegro moderato*) is linked to the preceding sections by additions of naturals to the new signature (a device not used elsewhere in the autograph).'

¹⁵ Previous analyses of works such as Opus 10, No.1 of 1798, Opus 31, No.2 of 1802, or Opus 73 of 1809 had revealed to me that Beethoven's dealings with proportions predate the writing of Opus 110. From the height of his experience (after 1919), it might have been easy for him, therefore, to perceive the plans conceived by Handel for the whole of his oratorio. My attempts to know which version of Messiah Beethoven examined have been unsuccessful. Burrows mentions (Op. cit., p.51) that Beethoven copied fragments in his sketch-books, but does not mention where they were copied from.

¹⁶ 'Blessed is he that cometh in the name of the Lord', as translated in the Dover edition.

¹⁷ 'Above the starry tent a loving Father must dwell', as translated in the score for the Ninth Symphony of the Dover edition 0-486-26035-6.

¹⁸ Example 2 (a, b, c) comes from Heinrich Schenker, Beethoven. Die letzten Sonaten. Sonate As dur Op. 110, Oswald Jonas ed. (Vienna, 1972). The sketches which show the reworking of the fugue are preserved as MS 51 in the Bibliothèque Nationale de Paris (Johnson, pp. 384-7).

¹⁹ The Table for Messiah in Appendix A shows that the metronome indications 60 and 90 (or the related 120 and 45) do not appear only in a few places where the character of the pieces might have impelled Gardiner (the conductor of the recorded version consulted) to look for a slight increase or decrease of speed. For Nos.40 and 41 he uses the crotchet at 135 (a bit faster than 120), possibly to allow the 'bravura aria' in No.40 to better express the 'furious rage' in the text. The crotchet at 105 (a bit slower than 120), used for Nos.42-44, contributes to the *Hallelujah's* magnificence. The crotchet at 75 (a bit slower than 90), used for Nos.49-51, besides favouring the solemn character required by the text in the chorus No.51, allows the intricacies of the double counterpoint in No.50 to be heard.

²⁰ Burrows study (Op. cit.), concerning the text and the librettist Charles Jennens's reactions during the initial phase of the oratorio's performance life, contains three items of data that must be considered. One is: 'Such evidence as there is suggests that Handel took an active role in his collaborations with oratorio librettists' (Burrows, p. 10)'. This statement supports my presumption that the composer might have influenced the librettist so that the text could serve an overall musical plan he already had in mind for Messiah. The other two pieces of data, referring to the period that followed the first performances, must be considered together: '(Jennens) had heard no performance and it is doubtful whether he had seen a score' (Burrows, p. 24), and 'I have said a great deal to him (Handel) on the Subject; but he is so lazy & so obstinate...' (from a Jennens's letter, in Burrows, p. 24). It would have been impossible for Jennens, without observing the score, to know how Handel had structured the tempos for organising the work's proportions. It is true that, revising the work to fit different performance circumstances, Handel used to alter the previous versions, and even insert or exclude a piece. He probably knew that such changes would be barely significant within the work as a whole (for example, No.38, *How beautiful are the feet*, according to Burrows, p. 19, was written in 1742; it occupies 0,44 centimetre in the 1/32 graph in Appendix B; the exclusion of No.38 would not greatly alter, therefore, the positioning of the main structural events). For the whole, however, it seems that the composer wanted to keep the control. This might have determined his omission in answering the demands for changes made by Jennens, who then came to qualify the composer as 'lazy and obstinate'.

²¹ The term 'prolongation' is used here as proposed by Heinrich Schenker in Free Composition, Ernst Oster transl. and ed. (New York, 1979). D major supports six choruses – Nos.9, 17, 35, 44, 48 and 53 – which act, in turn, as strong pillars of the whole construction. Different reasons can be suggested for the choice of D major. One perhaps related to the pitch A that the choral soprano voice can reach at the top of its range. By carrying the characteristic tension of the fifth degree in D tonalities, the pitch A becomes quite desirable when a melodic upper line is taken to its climax. Another reason can be found in the register of the baroque trumpet in D which could comfortably add brightness to the sound mass in tonic resolutions, or even play a solo role, as it does in the aria *The trumpet shall sound*. It would also be worth reflecting upon the affection the D major tonality was supposed to arouse, according to the baroque practice known as 'doctrine of affections.' Johann Mattheson is often associated with this subject because, by living in the eighteenth century, his writings document the contemporary practice. In our era, Claude Palisca in Baroque Music, 2nd ed. (Englewood Cliffs-NJ, 1981), pp.4-5, says the following on the subject: 'If, then, there is any common thread that unites the great variety of music that we call baroque, it is an underlying faith in music's power, indeed its obligation, to move the affections.' It is undeniable that in Messiah, D major perfectly fits the joy of the most glorious moments of the plot.

²² Announced by the imperative character of homophonic music, the polyphonic writing after m.[4] is enriched by augmentations, diminutions, double counterpoint, and elaborate techniques such as the one at m.[16]: the material in diminution which at m.[4] had integrated the fugal subject is given autonomy to become the subject of a new fugal event.

²³ 'The greatest work which I have composed' (Cooper, p.159).

²⁴ The making of the graph for Opus 123 departed from the idea that Beethoven was dealing with structured tempos. The *Kyrie* offers a good example. Between the initial *Assai sostenuto* and the *Andante assai ben marcato* of the *Christe*, at m.[86], a comfortable relationship is established, if the metronome is set for the minim, respectively, at 48 and 72.

²⁵ Among the sketches for the *Agnus Dei* the following remark by Beethoven is found: 'dona nobis pacem darstellend den innern und äussern Frieden' (delineating internal and external peace), as stated and translated by Thayer (Op. cit., p. 820).

²⁶ 'Beethoven approached the missal text chiefly with the imagination and the emotions of an artist, and that its poetical, not to say dramatic elements were those which he was most eager to delineate' (Ibid., p. 820).

²⁷ This conclusion derives from the comparative study of three different performances – under Gardiner (already mentioned), under Georg Solti (DECCA 444337-2), and under Carlo-Maria Giulini (EMI - CZS 7626932).

²⁸ See Jonathan del Mar's Critical Commentary on Opus 125 in the Bärenreiter edition of Beethoven's nine symphonies (Kassel, 1996).

²⁹ At this point, it seems appropriate to explain that the approach to 'time organisation', which became a priority in my analytical work, derives mostly from the contact I had with Brahms's rhythmic-metric intricacies for writing the dissertation 'Elements of Coherence in Brahms's Opus 76' (DMA Boston University, 1991), published in Brazil as Elementos de Coerência no Opus 76 de Brahms (Editora da UNICAMP, 1995).

³⁰ The analysis of Brahms's First was based on: Kalmus miniature score No.11747. The calculations in the table which appears in Appendix A derive from: 1) Epstein's suggestions in Beyond Orpheus (Op. cit., pp. 42-5), 2) the tempos adopted in recorded versions under Nikolaus Harnoncourt (TELDEC), Bruno Walter, and Zubin Mehta (for these versions, the reproductions I used did not provide references), and 3) my own understanding of the work. A thorough analysis of the means Brahms employs to organise the work's time would exceed the limits of the present study. Yet, the metronome numbers 48, 72 and 96 in the table are enough to clarify that the composer was dealing with structured tempos.